Laser Lecture 4 The Threshold Condition

University of Lucknow | Centenary Year

लखनऊ विश्वविद्यालय | शताब्दी वर्ष

100 Years 1920-2020

The Threshold Condition

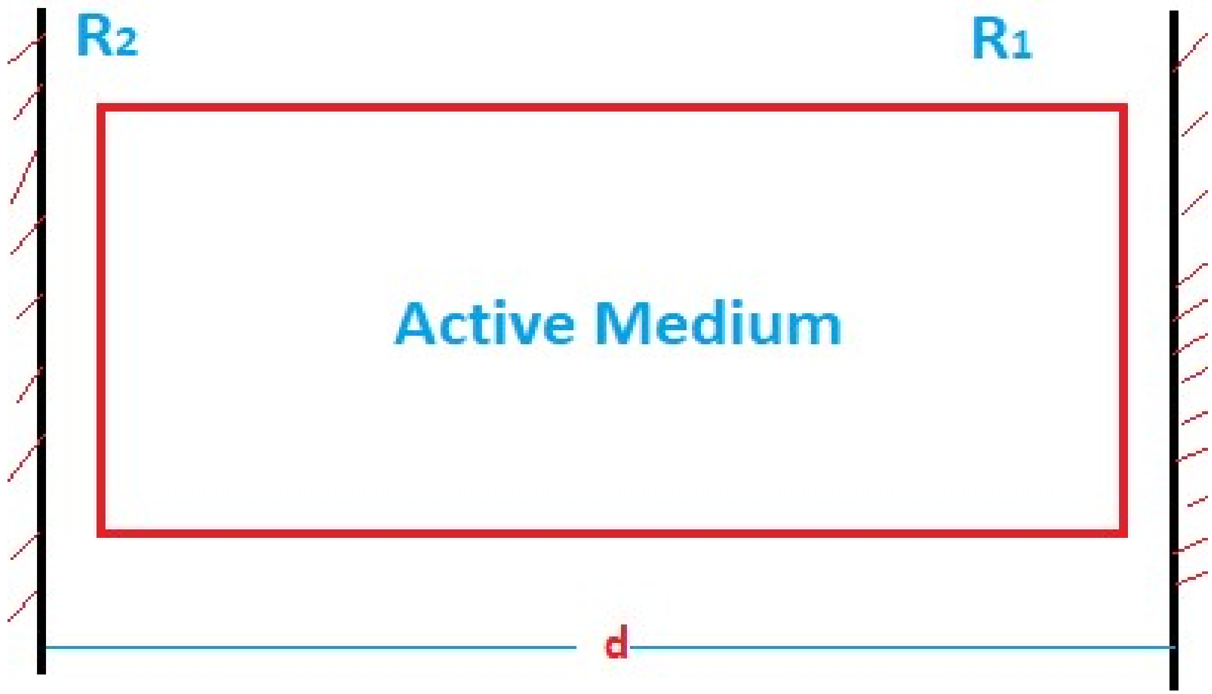
The laser threshold is the lowest excitation level at which a laser output is dominantly stimulated emission rather than spontaneous emission. For this a condition of some minimum population inversion must be maintained. Below the threshold, the laser output power grows gradually with increasing excitation or population inversion. The line width of the laser output becomes much smaller above the threshold than what it is below the threshold.

The threshold for laser oscillation is reached when the optical gain of the laser amplifying medium is exactly balanced by the sum of all the losses experienced by light in one round trip of the optical cavity resonator.

For a medium to be capable of amplifying an incident radiation, the state of population inversion must prevail in the medium

The medium will behave as an amplifier for those frequencies which fall within its line width

In order to generate radiation this amplifying medium is placed in an optical resonator which consists of a pair of mirrors facing each other



Radiation bouncing back and forth between the mirrors is amplified by the amplifying medium.

The radiation also suffers losses due to finite reflectivity of the mirrors and other diffraction and scattering losses.

For the oscillations to sustain in the cavity the losses must be compensated by gains.

Thus a minimum population inversion called the 'threshold population inversion density' is required to overcome the losses.

Expression for Threshold Population Inversion

d = length of the resonator; R_1, R_2 = reflectivity of mirrors

α_1 = average loss per unit length due to scattering, diffraction (due to finite size of mirrors), etc., other than finite reflectivity.

Consider a radiation of intensity **I** leaving mirror R_1

When this radiation propagating through the medium reaches the second mirror, it is amplified by $e^{-\alpha d}$ but at the same time suffers a loss of $e^{-\alpha_1 d}$ as well.

For an amplifying medium α is negative and $e^{-\alpha d} > 1$

Intensity of the beam after reflection at the second mirror will be $I R_2\, e^{-(\alpha_1+\alpha)d}$

Back journey of the beam from mirror R_2 to R_1 and reflection at mirror R_1 leads to an intensity of

$$I R_1 R_2\, e^{-2(\alpha_1+\alpha)d}$$

For laser oscillations to begin

$$R_1R_2\, e^{-2(\alpha_1+\alpha)d} \geq 1$$

The equality sign gives threshold value of α or the population inversion. Hence, if the laser is to oscillate in steady state with continuous wave the equality sign should be satisfied.

If the population inversion is increased beyond threshold LHS > 1.

In this case, the round trip gain becomes greater than the round trip loss. This leads to increase in intensity till saturation is achieved that would decrease the inversion.

$\Rightarrow$ $R_1R_2\, e^{-2(\alpha_1+\alpha)d} \geq 1$ $\Rightarrow$ $\ln R_1R_2 - 2(\alpha_1+\alpha)d \geq 0$

$\Rightarrow$ $(1/2d)\, \ln R_1R_2 \geq \alpha_1+\alpha$ $\Rightarrow$ $-\alpha \geq \alpha_1 - (1/2d)\, \ln R_1R_2$ (A)

The RHS of the equation depends on passive cavity parameters.

Passive Cavity Lifetime

The time in which the energy in the cavity reduces by a factor 1/e.

In the absence of amplifying medium ($\alpha = 0$), intensity at a point in one round trip is reduced by a factor

$$R_1R_2\, e^{-2\alpha_1 d} = e^{-(2\alpha_1 d - \ln R_1R_2)}$$

One round trip time is $t = \frac{2d}{c/n_o} = \frac{2dn_o}{c}$

Thus if the intensity reduces as $e^{\frac{-t}{t_c}}$

then in time $t = \frac{2dn_o}{c}$

the factor by which intensity will be reduced is $e^{\frac{-2dn_o}{ct_c}}$

$$e^{-(2\alpha_1 d - lnR_1R_2)} = e^{\frac{-2dn_o}{ct_c}}$$

$$\frac{1}{t_c} = \frac{c}{2dn_o}(2\alpha_1 d - lnR_1R_2) \qquad \frac{1}{t_c} = \frac{c}{n_o}\left(\alpha_1 - \frac{1}{2d}lnR_1R_2\right)$$

Use this expression for $1/t_c$ and expression for α on the side

$$\alpha = \frac{\pi^2 c^2}{n_o^2 \omega^2}\frac{1}{t_{sp}} g(\omega)\,(N_1 - N_2)$$

$$\Rightarrow N_2 - N_1 \geq \frac{4\nu^2 n_o^3}{c^3}\frac{t_{sp}}{t_c}\frac{1}{g(\omega)}$$

Thus for low threshold value of population inversion the following conditions should be satisfied:

- Value of t_c should be large
- Large value of $g(\omega)$ → small $\Delta\omega$
- Small value of t_{sp}

Threshold population inversion is proportional to ω^2. For small population inversion ω should be small. It is easier to obtain laser action at infra-red wavelength than in the ultra-violet region

YouTube Channel Link: https://www.youtube.com/channel/UC3rdRYA605bdDdsJdEf0oJw/featured

Prof. Narendra Kumar Pandey
Department of Physics
University of Lucknow
Lucknow-226007
Email: profnarendrapandey137@gmail.com